EXHIBIT D

LITERATURE REGARDING PLASMA QUASI-NEUTRALITY

INTRODUCTION TO

PLASMA PHYSICS AND CONTROLLED FUSION

SECOND EDITION

Volume 1: Plasma Physics

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No simplification is possible for the region near the grid, where $|e\phi/KT_e|$ may be large. Fortunately, this region does not contribute much to the thickness of the cloud (called a sheath), because the potential falls very rapidly there. Keeping only the linear terms in Eq. [1-13], we have

$$\epsilon_0 \frac{d^2\phi}{dx^2} = \frac{n_\infty e^2}{KT_e}\phi \qquad [1\text{-}14]$$

Defining

$$\boxed{\lambda_D \equiv \left(\frac{\epsilon_0 KT_e}{ne^2}\right)^{1/2}} \qquad [1\text{-}15]$$

where n stands for n_∞, we can write the solution of Eq. [1-14] as

$$\phi = \phi_0 \exp(-|x|/\lambda_D) \qquad [1\text{-}16]$$

The quantity λ_D, called the Debye length, is a measure of the shielding distance or thickness of the sheath.

Note that as the density is increased, λ_D decreases, as one would expect, since each layer of plasma contains more electrons. Furthermore, λ_D increases with increasing KT_e. Without thermal agitation, the charge cloud would collapse to an infinitely thin layer. Finally, it is the *electron* temperature which is used in the definition of λ_D because the electrons, being more mobile than the ions, generally do the shielding by moving so as to create a surplus or deficit of negative charge. Only in special situations is this not true (see Problem 1-5).

The following are useful forms of Eq. [1-15]:

$$\begin{aligned} \lambda_D &= 69(T/n)^{1/2}\ \text{m}, \qquad T \text{ in } {}^\circ\text{K} \\ \lambda_D &= 7430(KT/n)^{1/2}\ \text{m}, \qquad KT \text{ in eV} \end{aligned} \qquad [1\text{-}17]$$

We are now in a position to define "quasineutrality." If the dimensions L of a system are much larger than λ_D, then whenever local concentrations of charge arise or external potentials are introduced into the system, these are shielded out in a distance short compared with L, leaving the bulk of the plasma free of large electric potentials or fields. Outside of the sheath on the wall or on an obstacle, $\nabla^2\phi$ is very small, and n_i is equal to n_e, typically, to better than one part in 10^6. It takes only a small charge imbalance to give rise to potentials of the order of KT/e. The plasma is "quasineutral"; that is, neutral enough so that one can take $n_i \simeq n_e \simeq n$, where n is a common density called the *plasma*

5.7 THE SINGLE-FLUID MHD EQUATIONS

We now come to the problem of diffusion in a fully ionized plasma. Since the dissipative term $\mathbf{P}_{ei}$ contains the difference in velocities $\mathbf{v}_i - \mathbf{v}_e$, it is simpler to work with a linear combination of the ion and electron equations such that $\mathbf{v}_i - \mathbf{v}_e$ is the unknown rather than $\mathbf{v}_i$ or $\mathbf{v}_e$ separately. Up to now, we have regarded a plasma as composed of two interpenetrating fluids. The linear combination we are going to choose will describe the plasma as a single fluid, like liquid mercury, with a mass density ρ and an electrical conductivity $1/\eta$. These are the equations of magnetohydrodynamics (MHD).

For a quasineutral plasma with singly charged ions, we can define the mass density ρ, mass velocity $\mathbf{v}$, and current density $\mathbf{j}$ as follows:

$$\rho \equiv n_i M + n_e m \approx n(M + m) \qquad [5\text{-}78]$$

$$\mathbf{v} \equiv \frac{1}{\rho}(n_i M \mathbf{v}_i + n_e m \mathbf{v}_e) \approx \frac{M\mathbf{v}_i + m\mathbf{v}_e}{M + m} \qquad [5\text{-}79]$$

$$\mathbf{j} \equiv e(n_i \mathbf{v}_i - n_e \mathbf{v}_e) \approx ne(\mathbf{v}_i - \mathbf{v}_e) \qquad [5\text{-}80]$$

In the equation of motion, we shall add a term $Mn\mathbf{g}$ for a gravitational force. This term can be used to represent any nonelectromagnetic force applied to the plasma. The ion and electron equations can be written

$$Mn\frac{\partial \mathbf{v}_i}{\partial t} = en(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \nabla p_i + Mn\mathbf{g} + \mathbf{P}_{ie} \qquad [5\text{-}81]$$

$$mn\frac{\partial \mathbf{v}_e}{\partial t} = -en(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \nabla p_e + mn\mathbf{g} + \mathbf{P}_{ei} \qquad [5\text{-}82]$$

For simplicity, we have neglected the viscosity tensor $\boldsymbol{\pi}$, as we did earlier. This neglect does not incur much error if the Larmor radius is much smaller than the scale length over which the various quantities change. We have also neglected the $(\mathbf{v} \cdot \nabla)\mathbf{v}$ terms because the derivation would be unnecessarily complicated otherwise. This simplification is more difficult to justify. To avoid a lengthy discussion, we shall simply say that $\mathbf{v}$ is assumed to be so small that this quadratic term is negligible.

We now add Eqs. [5-81] and [5-82], obtaining

$$n\frac{\partial}{\partial t}(M\mathbf{v}_i + m\mathbf{v}_e) = en(\mathbf{v}_i - \mathbf{v}_e) \times \mathbf{B} - \nabla p + n(M + m)\mathbf{g} \qquad [5\text{-}83]$$

The electric field has cancelled out, as have the collision terms $\mathbf{P}_{ei} = -\mathbf{P}_{ie}$. We have introduced the notation

$$p = p_i + p_e \qquad [5\text{-}84]$$

for the total pressure. With the help of Eqs. [5-78]–[5-80], Eq. [5-83] can be written simply

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \mathbf{j} \times \mathbf{B} - \nabla p + \rho \mathbf{g} \tag{5-85}$$

This is the single-fluid equation of motion describing the mass flow. The electric field does not appear explicitly because the fluid is neutral. The three body forces on the right-hand side are exactly what one would have expected.

A less obvious equation is obtained by taking a different linear combination of the two-fluid equations. Let us multiply Eq. [5-81] by m and Eq. [5-82] by M and subtract the latter from the former. The result is

$$Mmn \frac{\partial}{\partial t}(\mathbf{v}_i - \mathbf{v}_e) = en(M + m)\mathbf{E} + en(m\mathbf{v}_i + M\mathbf{v}_e) \times \mathbf{B} - m \nabla p_i + M \nabla p_e - (M + m)\mathbf{P}_{ei} \tag{5-86}$$

With the help of Eqs. [5-78], [5-80], and [5-61], this becomes

$$\frac{Mmn}{e} \frac{\partial}{\partial t}\left(\frac{\mathbf{j}}{n}\right) = e\rho\mathbf{E} - (M + m)ne\eta\mathbf{j} - m \nabla p_i + M \nabla p_e + en(m\mathbf{v}_i + M\mathbf{v}_e) \times \mathbf{B} \tag{5-87}$$

The last term can be simplified as follows:

$$m\mathbf{v}_i + M\mathbf{v}_e = M\mathbf{v}_i + m\mathbf{v}_e + M(\mathbf{v}_e - \mathbf{v}_i) + m(\mathbf{v}_i - \mathbf{v}_e) = \frac{\rho}{n}\mathbf{v} - (M - m)\frac{\mathbf{j}}{ne} \tag{5-88}$$

Dividing Eq. [5-87] by $e\rho$, we now have

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} - \eta\mathbf{j} = \frac{1}{e\rho}\left[\frac{Mmn}{e}\frac{\partial}{\partial t}\left(\frac{\mathbf{j}}{n}\right) + (M - m)\mathbf{j} \times \mathbf{B} + m \nabla p_i - M \nabla p_e\right] \tag{5-89}$$

The $\partial/\partial t$ term can be neglected in slow motions, where inertial (i.e., cyclotron frequency) effects are unimportant. In the limit $m/M \to 0$, Eq. [5-89] then becomes

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta\mathbf{j} + \frac{1}{en}(\mathbf{j} \times \mathbf{B} - \nabla p_e) \tag{5-90}$$

FUSION PLASMA ANALYSIS

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plasma with a charged particle number density of $10^{20}\ m^{-3}$ and suppose that the electron number density (n_e) in a spherical volume of $10^{-3}\ m$ radius (r) were to differ by one per cent from the positive ion number density (n_i). Denoting the ion charge by e and the electron charge by $-e$, the total net charge (q) inside the sphere would be

$$q = \frac{4}{3}\pi r^3 (n_i - n_e) e \tag{2.1}$$

and the electric potential (ϕ) at the surface of the sphere would be

$$\phi = \frac{1}{4\pi\epsilon_o}\frac{q}{r} = \frac{r^2 e}{3\epsilon_o}(n_i - n_e) \tag{2.2}$$

where ϵ_o is the permittivity of free space. Plugging numerical values into (2.2) yields $\phi = 6 \times 10^3\ volts$. Recalling that $1\ eV = 1.602 \times 10^{-19}\ joule$, we find that $kT = 1\ eV$ when $T = 11,600\ K$, where k is Boltzmann's constant ($1.380 \times 10^{-23}\ joule/K$). Therefore, a plasma temperature of several millions of degrees Kelvin would be required to balance the electric potential energy with the average thermal particle energy.

Departures from macroscopic electrical neutrality can naturally occur only over distances in which a balance is obtained between the thermal particle energy, which tends to disturb the electrical neutrality, and the electrostatic potential energy resulting from any charge separation, which tends to restore the electrical neutrality. This distance is of the order of a characteristic length parameter of the plasma, called the *Debye length*. In the absence of external forces, the plasma cannot support departures from macroscopic neutrality over larger distances than this, since the charged particles are able to move freely to neutralize any regions of excess space charge in response to the large Coulomb forces that appear.

2.2 Debye Shielding

The *Debye length* is an important physical parameter for the description of a plasma. It provides a measure of the distance over which the influence of the electric field of an individual charged particle (or of a surface at some nonzero potential) is felt by the other charged particles inside the plasma. The charged particles arrange themselves in such a way as to effectively shield any electrostatic fields within a distance of the order of the Debye length. This shielding of electrostatic fields is a consequence of the collective effects of the plasma particles. A calculation of the shielding

than $\vec{W}$, and making use of Eq. 4-30, finally leads to

$$\rho\frac{\partial\vec{u}}{\partial t}+\rho(\vec{u}\cdot\nabla)\vec{u}+\nabla\cdot\vec{P}=\Sigma\vec{E}+\vec{j}\times\vec{B}+\vec{S}_\rho^1-S_\rho^0\vec{u}, \tag{4-38}$$

where

$$\vec{S}_\rho^1\equiv\sum_\sigma\vec{S}_\sigma^1. \tag{4-39}$$

The collisional friction term does not appear in Eq. 4-38 because $\Sigma_\sigma\vec{R}_\sigma^1=0$ from momentum conservation in elastic collisions.

To obtain a one-fluid equation for the current density, we multiply Eq. 4-7 by e_σ/m_σ and sum over species. Using the velocity decomposition of Eq. 4-37, defining the pressure tensor in terms of $\vec{y}$ instead of $\vec{W}$, and making use of the fact that $m_e/m_\sigma\ll 1$ for $\sigma\neq e$ leads to

$$\frac{m_e}{n_e e_e^2}\left[\frac{\partial\vec{j}}{\partial t}+\nabla\cdot(\vec{u}\vec{j}+\vec{j}\,\vec{u}-\Sigma\vec{u}\vec{u})\right]+\frac{1}{n_e e_e}\nabla\cdot\vec{P}_e$$
$$=\left[\vec{E}+\left(1-\frac{\Sigma}{n_e e_e^2}\right)\vec{u}\times\vec{B}+\frac{1}{n_e e_e}\vec{j}\times\vec{B}\right]-\eta\vec{j}+\frac{m_e}{n_e e_e^2}\vec{S}_q^1, \tag{4-40}$$

where

$$\eta\vec{j}\equiv-\frac{m_e}{n_e e_e^2}\sum_\sigma e_\sigma\int\vec{v}C_\sigma d^3v, \tag{4-41}$$

$$\vec{S}_q^1\equiv\sum_\sigma\frac{e_\sigma}{m_\sigma}\vec{S}_\sigma^1. \tag{4-42}$$

The quantity η defined by Eq. 4-41 is the plasma resistivity. The subscript e refers to electrons.

Equations 4-38 and 4-40 can be simplified further when the length (L) and time (τ) scales of interest are large compared to certain characteristic parameters of the plasma. Since the potential of a charge is Debye-shielded, the plasma is effectively neutral on a length scale $L\gg\lambda_D$, so that the quasineutrality approximation, $\Sigma\to 0$, can be used.

In many situations of practical interest, it can be shown that the distribution function is an isotropic Maxwellian plus a small correction on the order of $\delta\equiv r_{Li}/L$. In such cases, since the RHS of Eq. 4-4 vanishes for an isotropic distribution, $\vec{v}_\sigma$ and $\vec{u}$ are of first order in δ. Multiplying Eq. 4-38 by L/ρ, the

C. MAGNETOHYDRODYNAMIC MODEL

A specialization of the one-fluid model that is frequently used, together with Maxwell's equations, for the analysis of plasmas is known as the magnetohydrodynamic (MHD) model. The MHD model consists of

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{u} = 0, \qquad \text{continuity;} \tag{4-49}$$

$$\rho \frac{\partial \vec{u}}{\partial t} + \nabla p = \vec{j} \times \vec{B}, \qquad \text{momentum;} \tag{4-50}$$

$$\eta \vec{j} = \vec{E} + \vec{u} \times \vec{B}, \qquad \text{Ohm's law;} \tag{4-51}$$

$$\left.\begin{aligned} \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \mu_0 \vec{j} &= \nabla \times \vec{B} \end{aligned}\right\}, \qquad \text{Maxwell's equations;} \qquad \begin{aligned}(4\text{-}52)\\(4\text{-}53)\end{aligned}$$

and a suitable "equation of state" instead of the energy balance equation such as

$$\frac{d}{dt}\left(\frac{p}{\rho^{5/3}}\right) = 0, \qquad \text{adiabatic;} \tag{4-54}$$

or

$$\nabla \cdot \vec{u} = 0, \qquad \text{incompressible;} \tag{4-55}$$

or

$$\frac{d}{dt}\left(\frac{p}{\rho}\right) = 0, \qquad \text{isothermal.} \tag{4-56}$$

From our previous derivation of the one-fluid equations, we can conclude that certain assumptions are implicit in the MHD model. The use of a scalar pressure assumes an isotropic distribution to lowest order in $\delta \equiv r_{Li}/L$. A source-free problem is obviously assumed. Neglect of Σ implies that the model is valid for spatial scales $L \gg \lambda_D$, and neglect of certain other terms
p. 83 indicates that $L \gg r_{Li}$ is a requirement for validity. The use of a one-fluid model implies that the electrons and ions respond together. This implies the model is valid on time scales that are long compared to the inverse of the lowest of the plasma characteristic frequencies discussed in Chapters 1 and 2, which is the ion gyrofrequency; that is, $\tau \gg \Omega_i^{-1}$. In fact, the MHD ordering is

$$1 \gg \frac{1}{\tau \Omega_i} \gg \delta^2. \tag{4-57}$$

FUNDAMENTALS OF PLASMA PHYSICS

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fusion research.

Another important aspect of plasma behavior is the emission of *radiation.* The main interest in plasma radiation lies in the fact that it can be used to infer plasma properties. The mechanisms that cause plasmas to emit or absorb radiation can be grouped into two categories: radiation from emitting atoms or molecules, and radiation from accelerated charges. At the same time that ionization is produced in a plasma, the opposite process, *recombination* of the ions and electrons to form neutral particles, is normally also occurring. As a result of the recombination process, radiation is often emitted as the excited particles formed during recombination decay to the ground state. This radiation constitutes the *line spectra* of plasmas. On the other hand, any accelerated charged particle emits radiation. The radiation emitted whenever a charged particle is decelerated by making some kind of collisional interaction is called *bremsstrahlung.* If the charged particle remains unbound, both before and after the encounter, the process is called *free-free bremsstrahlung.* Radiation of any wavelength can be emitted or absorbed in bremsstrahlung. If the originally unbound charged particle is captured by another particle, as it emits the radiation, the process is called *free-bound radiation. Cyclotron radiation,* which occurs in magnetized plasmas, is due to the magnetic centripetal acceleration of the charged particles as they spiral about the magnetic field lines. *Blackbody radiation* emitted from plasmas in thermodynamic equilibrium is important only in astrophysical plasmas, in view of the large size needed for a plasma to radiate as a blackbody.

2. CRITERIA FOR THE DEFINITION OF A PLASMA

2.1 Macroscopic Neutrality

In the absence of external disturbances a plasma is *macroscopically neutral.* This means that under equilibrium conditions with no external forces present, in a volume of the plasma sufficiently large to contain a large number of particles and yet sufficiently small compared to the characteristic lengths for variation of macroscopic parameters such as density and temperature, the *net* resulting electric charge is zero. In the interior of the plasma the microscopic space charge fields cancel each other and no net space charge exists over a macroscopic region.

If this macroscopic neutrality was not maintained, the potential energy associated with the resulting Coulomb forces could be enormous compared to the thermal particle kinetic energy. Consider, for example, a